DYNAMIC TRUNK RESERVATION FOR TELETRAFFIC LINKS

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Abstract — A state dependent cost of accepting a call is introduced to measure the benefit of alternate routes. This cost function, which requires expensive calculations, can be approximated to a linear form without much error. Based on this cost approximation, a dynamic trunk reservation level is derived. It is easy to implement on-line and gives a better overload performance than a fixed trunk reservation level. Simulation studies confirm our results.

I INTRODUCTION

Recent advances in Stored Program Control switching systems offer telephone networks improved flexibility in traffic control and management. With this capability, the network cost required to maintain a certain level of blocking performance in dynamic non-hierarchical routing networks is less than that in fixed hierarchical routing networks [1].

However non-hierarchical routing is found to be unstable under overload conditions. In 1973, Nakagome and Mori [2] first reported on theoretical studies of the instabilities in symmetric alternate routing networks. Krupp [3] presented simulation results which exhibited bistable behavior. He also demonstrated that the high blocking probability is caused by excessive use of alternate routes, and that reserving a small number of trunks for direct-routed traffic in each group would stabilize the network and prevent degradation of performance under overload. Later, Akinpelu [4] extended the analysis to more general non-symmetric networks and found that similar types of network instabilities exist. She also proposed an empirically determined 5% trunk reservation level.

A simple inference can show that the fixed trunk reservation may not be optimal. More trunks should be reserved to protect the direct routed calls under heavy traffic, while a lower trunk reservation level under lightly loaded conditions can offer more alternate routes which reduce blocking. That is, the optimal trunk reservation level should depend on traffic load. Cameron first realized this and proposed a dynamically adjusted protective allowance based on the overflow measurements [5]. Later Yum and Schwartz put forward an external blocking scheme [6] which simply switched off alternate routes at heavy traffic load.

The foregoing results are basically heuristic: we know that trunk reservation can be beneficial, but the best level of reservation has to be determined experimentally. One might even say that trunk reservation falls into the category of "clever tricks" which are known to work in practice even though there is little supporting theory. In a recent paper [7] Mitra et al derived a load dependent trunk reservation level by asymptotic analysis of the Fixed Point Model [8].

In this paper, we explore a theoretical basis for deciding when to accept alternate-routed calls, based on deriving a formula for the state dependent cost of accepting a call. Here "state" refers to the occupancy of a network link, which is a common concept in telecommunication engineering. The optimal decisions based on this cost formula would require expensive calculations, but we argue that a linear approximation to the cost gives results which are not too far removed from the optimum. With this approximation, it turns out that our results can be expressed in the language of dynamic trunk reservation. That is, the near-optimal decision on whether to accept an alternate-routed call is based on comparing the number of free trunks with a threshold value; and we are able to give a simple formula for computing that threshold.

II STATE DEPENDENT COST AND ITS APPROXIMATION

To provide a measure of the performance of a network link in operation, let each call be assigned a "worth". If the revenue from the link is the worth of total calls completed, then the cost of accepting a call is the worth of all the calls that are lost as a consequence of carrying that call on the link. The idea of translating call acceptance into the concepts of revenue and cost was first proposed by F.P. Kelly. In [9] the cost was interpreted as Erlang's improvement formula,

$$\rho[EB(\rho, N-1) - EB(\rho, N)]$$

that is, the increase in the expected number of lost calls per unit time if a single circuit is removed from the link. This load dependent cost function is fundamental to Kelly's routing analysis.

Realizing that the number of lost calls on a link is also dependent on the link occupancy, we try to define and calculate a state dependent cost function. In the following, we will deal with $B(t)$ the cumulative count of blocked calls on a teletraffic link with call arrival rate λ and departure rate μ. Suppose at time $t = 0$, a call comes to a link with capacity N and occupancy $Z(0) = l$, $l < N$. The blocked call count for this link will start with $E[B(0)|Z(0) = l+1] = 0$; on the other hand, if this incoming call is rejected (not blocked), the blocked call count will start with $E[B(0)|Z(0) = l] = 0$.

Assume that during a time period $(0, t]$, the number of call arrivals is equal to the number of call departures on that link, that is

$$Z(t) = Z(0)$$

this is true for a stationary queue, in particular for large t. At time t, the expected number of blocked call counts for the above two cases will be $E[B(t)|Z(t) = l+1] = E[B(t)|Z(0) = l+1]$ and $E[B(t)|Z(t) = l] = E[B(t)|Z(0) = l]$ respectively. And their difference is the expected number of additional calls blocked if we accepted the incoming call at $t = 0$,

$$E[B(t)|Z(t) = l+1] - E[B(t)|Z(t) = l] \qquad (1)$$

It is expected that the first term in Equation (1) will increase faster than the second term during the holding period of the additional call. After that the two increments will be the same. This means that the transient term of Equation (1) is significant when t is comparable with the mean holding time of that call. In order to calculate the steady state value of Equation (1), we need the following lemma,

Lemma 1. *[10]*

$$0 < \lim_{t\to\infty}\{E[B(t)|Z(t) = l+1] - E[B(t)|Z(t) = l]\} < \infty \quad (2)$$

Definition. *The state dependent cost $C^s(l,\rho)$ of accepting a call on a teletraffic link with call density ρ ($= \lambda/\mu$) and occupancy l is defined as the steady state value of the expected number of additional calls blocked on that group as a consequence of carrying the present incoming call.*

$$C^s(l,\rho) \triangleq \lim_{t\to\infty}\{E[B(t)|Z(t) = l+1] - E[B(t)|Z(t) = l]\} \quad (3)$$

For a teletraffic network, under a standard independent assumption [3], [4], [11], the overall cost of accepting a call is, of course $\sum_g C^s_g(l,\rho)$, where the sum is taken over all links involved in connecting the call.

It should be pointed out that this state dependent cost is similar to Krishnan's relative cost. In [12], the cost was derived by examining the state transition of a Poisson process. Here our state dependent cost is derived through the dynamic analysis of an $M/M/N/N$ queue, and has a clear statistical expression. In order to calculate the cost, we need the following result.

Lemma 2. *[10]*

$$\lim_{t\to\infty}\{(l+1)\mu p_{l+1}(t)E[B(t)|Z(t) = l+1] - \lambda p_l(t)E[B(t)|Z(t) = l]\} = \frac{\lambda p_N}{\sum_{i=0}^{N}\frac{(\lambda/\mu)^i}{i!}}\sum_{i=0}^{l}\frac{(\lambda/\mu)^i}{i!} \quad (4)$$

With this result, our state dependent cost can be calculated by the following Theorem.

Theorem 3. *For a teletraffic link, the state dependent cost defined in Equation (3) is:*

$$C^s(l,\rho) = \frac{p_N}{EB(\rho,l)} \qquad (5)$$

where p_N is the blocking probability of that trunk group, and $EB(\rho,l)$ is the Erlang-B formula.

The proof of this Theorem is given in the Appendix.

The advantage of this cost function is that it is both load and state dependent. In Fig. 1 (a) and (b), $C^s(l,\rho)$ are plotted versus link state l in solid lines for $N = 10$ and $N = 100$ respectively under several offered traffic densities ρ. It can be seen that the cost is higher when occupancy is higher and/or traffic is heavy.

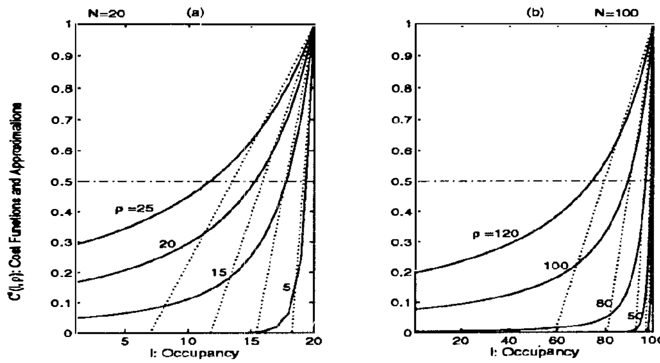

Figure 1: State Dependent Cost and Approximation

The difficulty with Equation (5) is that $EB(l,\rho)$ is not directly measurable and is quite time consuming to calculate. Moreover the blocking probability p_N is difficult to measure on-line; and, without this measurement, more computational effort will be involved. In order to obtain the state dependent cost $C^s(l,\rho)$ on-line, approximations have to be made to get a computationally effective form. Observe that all the cost functions are near linear above the 0.5 line, it is tempted to use linear approximation for this part of the curves. It will be shown later that fortunately only this part of cost functions are needed in determining trunk reservation level.

In order to make the linear approximation we need one point and one slope. At $l = N$, we have $EB(\rho,N) = p_N$, so that $C^s(l,\rho)|_{l=N} = 1$ is naturally a good candidate for the point. For the slope, we need the derivative $D^s(l,\rho)$ of the cost which is given by the following corollary.

Corollary 4. *[10]*

$$D^s(l,\rho) \triangleq \frac{\partial C^s(l,\rho)}{\partial l} \approx (1 - \frac{\rho}{l})C^s(l,\rho) + \frac{p_N\rho}{l} \qquad (6)$$

The slope at the end-point equals the derivative at $l = N$, and this simplifies to

$$D^s(N,\rho)|_{l=N} = \frac{N-(1-p_N)\rho}{N}$$

With the above end-point and slope, we obtain a linear approximation $\hat{C}^s(l,\rho)$ for the cost function $C^s(l,\rho)$ as:

$$\hat{C}^s(l,\rho) \approx 1 - \frac{N-(1-p_N)\rho}{\theta N}(N-l) \qquad (7)$$

where $\theta \geq 1$ is a factor chosen to adjust the slope to make a closer approximation. Numerical studies suggest that

$\theta = 1 + 0.1 ln N$ is appropriate. The approximations for the above example of cost functions are shown in Fig.1 in dotted lines.

III DYNAMIC TRUNK RESERVATION LEVEL

It is easy to check that the cost $C^s(l, \rho)$ for carrying a call on a trunk group is always less or equal to one. This means that it is always of benefit to accept and complete an on coming call on the trunk group if there are free trunks available. This is true for direct-routed calls. For alternate-routed calls which occupy two or more links, the cost is the sum over every link used, and this may be larger than one. In this case, it means that by carrying the incoming call on an alternate route, the sum of the expected number of calls blocked on each link may be more than one. At this stage, alternate routes are doing more harm than good to the network, and they should be closed by trunk reservation schemes.

The problem with this criterion is that it depends on looking at more than one trunk group. For the sake of a simple implementation, a local criterion may be preferable. Let us therefore define a cost threshold $\eta < 1$, such that whenever the cost on a trunk group exceeds this threshold, free trunks will be reserved for direct-routed call use only. In this way, the state l_s at which trunk reservation should be triggered is determined via,

$$C^s(l_s, \rho) = \eta \tag{8}$$

In a network with two link alternate routes, which is the case in many dynamic non-hierarchical networks, an alternate-routed call will worth half of a direct-routed call on each link. So it seemed that η should be around 0.5.

As an example, consider the symmetrical, fully connected and uniformly loaded network analyzed in [3] and [6]. Under an independence assumption, given a preassigned trunk reservation level r, network dynamics can be obtained by solving the following nonlinear algebraic equations — the Fixed Point Model,

$$A = \rho_e\{1 + \frac{2p_N}{x_r}[1 - (1 - x_r^2)^m]\} \tag{9a}$$

$$p_N = \frac{A^{N-r}\rho_e^r}{N!} p_0 \tag{9b}$$

$$x_r = \sum_{i=0}^{N-r-1} \frac{A^i}{i!} p_0 \tag{9c}$$

where

$$p_0 = \left(\sum_{i=0}^{N-r} \frac{A^i}{i!} + \sum_{i=N-r+1}^{N} \frac{A^{N-r}\rho_e^{i-N+r}}{i!}\right)^{-1}$$

Here ρ_e is the end-to-end offered load, and A is the link offered load which includes both original and overflowed traffic. N is the link capacity and m denotes the number of two link alternate paths that a call can try before being blocked or rejected. p_N, p_0, and x_r are the probabilities of link blocking, empty and below the reservation level, respectively.

Network performance is defined as the end-to-end blocking probability $EEBP$,

$$EEBP = p_N(1 - x_r^2)^m$$

For a given end-to-end offered load ρ_e and a fixed trunk reservation level r, network performance $EEBP$ can be calculated by the above procedure. Let the optimal trunk reservation level r^* be defined as that at which best performance is obtained,

$$r^*(\rho_e) \triangleq \{r : \min_r EEBP(\rho_e, r)\}$$

In Fig.2 (a) and (b), r^* are evaluated numerically for Network A ($N = 20, \quad m = 10$) and Network B ($N = 100, \quad m = 10$) under various load conditions.

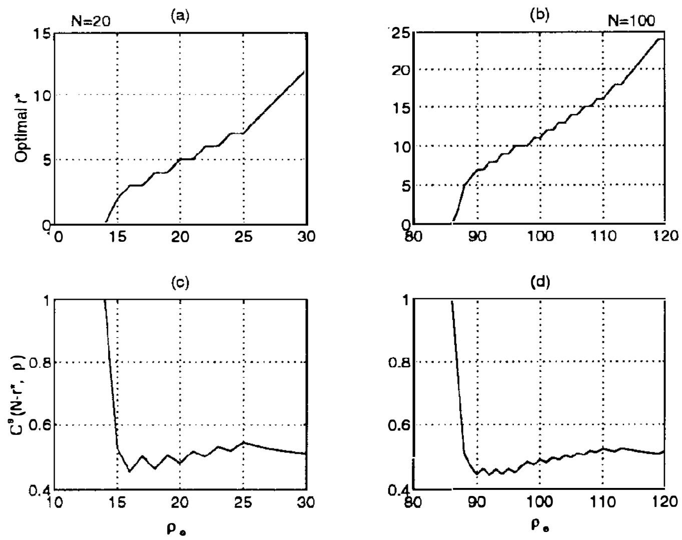

Figure 2: Link Cost at Optimal Trunk Reservation Level

The cost thresholds at these optimal trunk reservation levels are also calculated,

$$C^s(l, \rho)|_{l=N-r^*} = C^s(N - r^*, \rho)$$

Here, ρ is the average link offered load,

$$\rho = x_r A + (1 - x_r)\rho_e$$

Fig.2 (c) and (d) shows that these thresholds are asymptotically 0.5 when load is heavy. This result confirmed our cost analysis above. When the load is light, the cost thresholds are higher. This can be understood as that at light load, the average link costs stay at lower level, and even when the cost of one link on an alternate route equals to 0.5, the cost of the other link on that route is often less. So the sum of the two cost is usually less than 1 which means that this alternate route is still beneficial for use. At this circumstance, the cost threshold η should be raised higher to accept such beneficial calls. It is expected that other aspects such as routing algorithms and network structures etc would also have some effects on the value of η. Numerical studies showed that it is not sensitive to changing network conditions, and the optimal η is around 0.6.

The problem in using Equation (8) is that the cost $C^s(l_s, \rho)$ is an implicit function of l_s in addition to the computational complexity and measurement difficulties. So our approximation $\hat{C}^s(l, \rho)$ is used instead of $C^s(l, \rho)$. Applying Equation (7) in the judgment Equation (8), we obtain the dynamic trunk reservation level r_S as:

$$r_S = N - l_s = \frac{(1-\eta)\theta}{N - (1-p_N)\rho} N \qquad (10)$$

It is interesting to note that $(1-p_N)\rho = E[Z(t)]$ is actually the expected number of calls in progress on that trunk group, and $Z(t)$ the occupancy usually plays the role of "state" in many adaptive routing schemes [5], [7], [8]. So that for practical implementation in such cases, no additional measurement is needed, and the computation involved in Equation (10) isn't difficult for on-line evaluation of the dynamic trunk reservation level.

IV PERFORMANCE COMPARISIONS

In the following, performances of both fixed and dynamic trunk reservation schemes are studied on Network A ($N = 20$, $m = 10$) and Network B ($N = 100$, $m = 10$). For the fixed trunk reservation scheme, given the end-to-end offered load ρ_e and trunk reservation level $r_F = 5\%N$, network states can be obtained by solving Equations (9a)-(9c) recursively. With our dynamic trunk reservation scheme, the trunk reservation level Equation (10) is added to the set of network equations. And r_S can be solved together with other parameters.

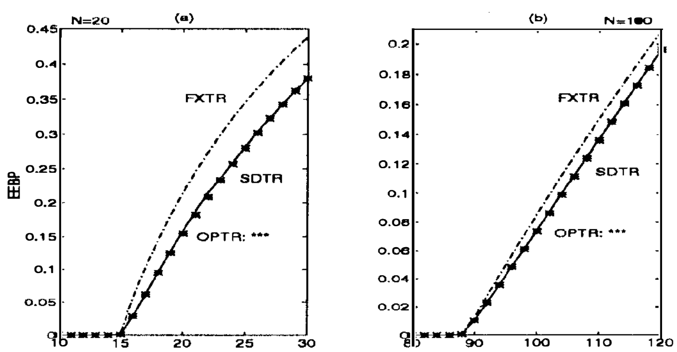

Figure 3: Performance Comparisons

In Fig.3 (a) and (b), the performances of network A and network B are evaluated respectively. Our dynamic trunk reservation scheme (SDTR — solid lines) is compared with the fixed scheme (FXTR — dash-dotted lines) under various load conditions. The performances under optimal trunk reservation levels (OPTR — stars) calculated numerically are also shown. The overload performance of SDTR is found to be much better than FXTR, and is near optimal.

A detailed performance comparison of the two schemes with the optimal one are shown in Fig.4. In (a) and (b),

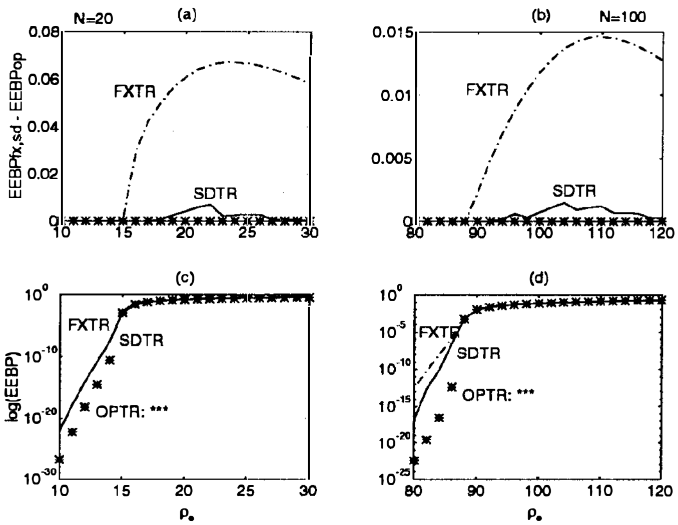

Figure 4: Detailed Performance Plots

the performance differences between SDTR and OPTR are plotted in solid lines, the performance differences between FXTR and OPTR are plotted in dash-dotted lines. It can be seen that the dynamic scheme is superior over the fixed one under overload conditions, and its performance is very close to the optimal case. When load is light, there isn't much difference in the performance measures, since all the end-to-end blocking probabilities are nearly zero. If we change the scale in plotting the performances, e.g. use logarithmic plot in Fig.4 (c) and (d), the differences in this light load area can be spotted. The performance of the dynamic scheme, although not as close to the optimal one as in the overload cases, is still not worse than the fixed one.

Various trunk reservation schemes are simulated [13] on a 12 node network ($N = 20$, $m = 10$). We compared our dynamic scheme (SDTR) with the fixed scheme (FXTR) and no reservation scheme (NOTR). Their performances are plotted in dots in Fig.5. The theoretical performances calculated by the above procedures are also plotted in lines. Their consistency with the simulated data confirmed our results. It turned out that the fixed trunk reservation level can only be tuned to suit one traffic level, while our dynamic scheme takes care of all traffic conditions. The near optimal performance under overload conditions makes the dynamic trunk reservation scheme an outstanding algorithm for overload control.

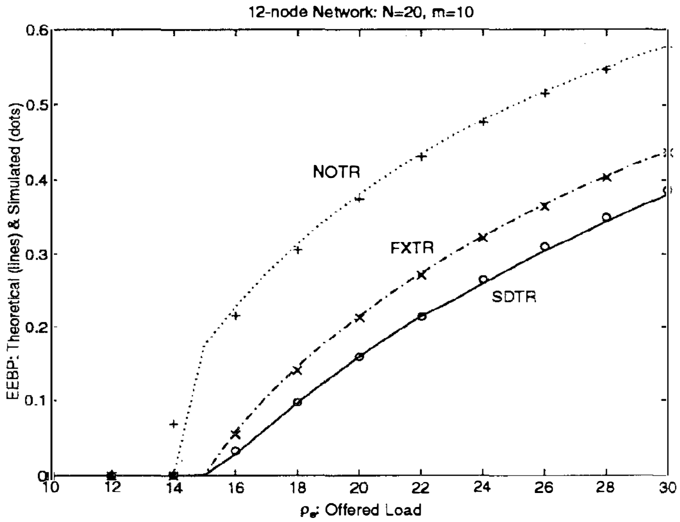

Figure 5: Simulation Studies

Appendix. PROOF OF THEOREM 3

Multiply the cost function Equation (3) by λp_l,

$$\begin{aligned} &\lambda p_l C^s(l,\rho) \\ &= \lim_{t\to\infty} \lambda p_l(t) \cdot \lim_{t\to\infty} \{E[B(t)|Z(t)=l+1] - E[B(t)|Z(t)=l]\} \\ &= \lim_{t\to\infty} \lambda p_l(t)\{E[B(t)|Z(t)=l+1] - E[B(t)|Z(t)=l]\} \end{aligned}$$

Comparing this with Equation (4), we have

$$\begin{aligned} &\lim_{t\to\infty}\{(l+1)\mu p_{l+1}(t)E[B(t)|Z(t)=l+1] - \lambda p_l(t)E[B(t)|Z(t)=l]\} \\ &-\lim_{t\to\infty}\{\lambda p_l(t)E[B(t)|Z(t)=l+1] - \lambda p_l(t)E[B(t)|Z(t)=l]\} \\ &= \lim_{t\to\infty}\{(l+1)\mu p_{l+1}(t)E[B(t)|Z(t)=l+1] \\ &\quad -\lambda p_l(t)E[B(t)|Z(t)=l+1]\} \\ &= \lim_{t\to\infty}\{[(l+1)\mu p_{l+1}(t) - \lambda p_l(t)]E[B(t)|Z(t)=l+1]\} \\ &= \lim_{t\to\infty} \frac{E[B(t)|Z(t)=l+1]}{\frac{1}{(l+1)\mu p_{l+1}(t)-\lambda p_l(t)}} \\ &= \lim_{t\to\infty} \frac{\lambda p_N + \text{transients}}{\frac{2\lambda^2 p_l(t)}{[(l+1)\mu p_{l+1}(t)-\lambda p_l(t)]^2}} \\ &= 0 \end{aligned}$$

This result means that

$$\begin{aligned} \lambda p_l C^s(l,\rho) &= \lim_{t\to\infty}\{(l+1)\mu p_{l+1}(t)E[B(t)|Z(t)=l+1] \\ &\quad -\lambda p_l(t)E[B(t)|Z(t)=l]\} \\ &= \frac{\lambda p_N}{\sum_{i=0}^{N}\frac{(\lambda/\mu)^i}{i!}} \sum_{i=0}^{l}\frac{(\lambda/\mu)^i}{i!} \end{aligned}$$

then we have

$$C^s(l,\rho) = \frac{p_N}{\frac{\frac{(\lambda/\mu)^l}{l!}}{\sum_{i=0}^{l}\frac{(\lambda/\mu)^i}{i!}}} = \frac{p_N}{EB(\rho,l)} \qquad (11)$$

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