STRUCTURE RESULT FOR NONLINEAR PASSIVE SYSTEMS

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Abstract

A class of nonlinear, finite-dimensional, dynamic systems is studied for which the input and output vectors u and y satisfy a passivity condition. It is shown that such systems may be viewed as a cascade of a memoryless passive nonlinear system and a dynamic lossless system. The discussion is related to known results on linear network synthesis.

1. INTRODUCTION

Classical network synthesis, for linear, lumped, finite, passive networks, is concerned with the problem of passing from a port description of a network in terms of, say, a positive real impedance matrix $Z(s)$, to a collection of (passive) network elements and a scheme for interconnecting them to produce a network with impedance matrix equal to that prescribed, [1-3]. State-space approaches to the same problem [4] commence by assuming known a state-variable realization $\{F, G, H, J\}$ (generally minimal) of $Z(s)$ - thus

$$Z(s) = J + H'(sI-F)^{-1}G \tag{1}$$

and then attempt to construct from this an internally dissipative realization $\{\bar{F}, \bar{G}, \bar{H}, J\}$, i.e. one for which

$$\begin{bmatrix} J+J' & -(\bar{H}-\bar{G})' \\ -(\bar{H}-\bar{G}) & -(\bar{F}+\bar{F}') \end{bmatrix} \geq 0 \tag{2}$$

Equivalently, one needs to find a positive definite P such that

$$\begin{bmatrix} J+J' & -(H-PG)' \\ -(H-PG) & -PF-F'P \end{bmatrix} \geq 0 \tag{3}$$

Once (2) or (3) is obtained, it is then possible to define easily a nondynamic coupling network N_c, synthesisable merely with (passive) resistors, transformers and gyrators, such that termination of some of the ports of N_c in inductors leads to an impedance $Z(s)$ being observed at the remaining ports. For details, see [4].

Our purpose here is to describe how some of these results will carry over to a nonlinear situation. Consider a system described by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) + j(x)u \end{aligned} \tag{4}$$

where $u(\cdot)$ and $y(\cdot)$ are real m-vector functions of time, $x(\cdot)$ is a real n-vector function of time and $f(\cdot)$, $g(\cdot)$, $h(\cdot)$, $j(\cdot)$ are suitably smooth real functions of appropriate dimension, with $f(0) = 0$, $h(0) = 0$. We call such a system passive if for all $u(\cdot)$ and t_1, given $x(t_0) = 0$, one has

$$\int_{t_0}^{t_1} u'y \, dt \geq 0 \tag{5}$$

This definition is a natural extension to that applying in the linear case; one can think of

$u(\cdot)$ and $y(\cdot)$ as current and voltage vectors respectively, so that the integral in (5) constitutes the energy input to a network with port variables $u(\cdot)$ and $y(\cdot)$, computed over $[t_0, t_1]$, and with the network initially unexcited.

Our task is to provide a nonlinear internally passive synthesis for (4). That is, we wish to find a nonlinear, nondynamic or memoryless, passive coupling network together with nonlinear passive inductors so that the arrangement depicted in Figure 1 (coupling network loaded at some ports by inductors) has u related to y by (4). Note that while (5) is a passivity condition on the network of Figure 1, it is an external one, directly putting constraints on the port behaviour alone of the network, and not the behaviour of internal variables nor the properties of components within the network.

Note also that our specification of both the nonlinear inductor network and the nondynamic coupling network resulting from the synthesis procedure will be simply via port descriptions of these networks - we shall not attempt to describe how to undo any mutual coupling of the nonlinear inductors for example. Accordingly, the contribution to practical network synthesis is virtually nil; the result is more one concerning the theory of passive systems, with electrical networks providing one means of visualizing the results.

In section 2, we present background results drawn from [5] which allow reinterpretation of the condition (5) in terms of the state-variable equations (4). These results are used in section 3 to present a passive synthesis. Section 4 contains concluding remarks.

2. BACKGROUND

Returning to the linear problem for the moment, we note that it is possible to associate with a passive (or positive real) $Z(s)$ in (1) a variational problem, the solution to which defines a positive definite P satisfying (3). It turns out that the same sort of idea can be employed in studying the passivity of (4).

Following [5], we shall assume that (4) is completely controllable in the sense that for any finite states x_0 and x_1, there exists a finite time t_1 and a smooth control defined on $[0, t_1]$ such that the state can be driven from $x(0) = x_0$ to $x(t_1) = x_1$. Further, we assume a form of local controllability: for any x_0 and any x_1 in a suitably small open neighbourhood of x_0, there exists a $u(\cdot)$ and t_1 as above with the additional property that

$$\left| \int_0^{t_1} u'(t)y(t)dt \right| \leq \rho(||x_1 - x_0||) \tag{6}$$

for some continuous $\rho(\cdot)$ such that $\rho(0) = 0$. (This equation in effect demands that changes of state must not use arbitrarily large amounts of energy). The main theorem of [5] then states that a necessary and sufficient condition for (5) to hold is that there should exist real functions $P(\cdot)$, $\ell(\cdot)$ and $w(\cdot)$ with $P(x)$ continuous and with, for all x,

$$P(x) \geq 0 \quad \text{and} \quad P(0) = 0 \tag{7}$$

$$\begin{aligned} f'(x)\nabla P(x) &= -\ell'(x)\ell(x) \\ \tfrac{1}{2}g'(x)\nabla P(x) &= h(x) - w'(x)\ell(x) \\ j(x) + j'(x) &= w'(x)w(x) \end{aligned} \tag{8}$$

These equations generalize those applying in the linear case, [4]. The results of the linear case are recovered by setting $f(x) = Fx$, $g(x) = G$, $h(x) = H'x$, $j(x) = J$ and $P(x) = x'Px$. The variational problem used in the linear case when translated to the nonlinear case yields the following characterization for one of the functions $P(x)$ satisfying (8):

$$P(x) = -\liminf_{\substack{T\to\infty \\ u(\cdot)}} \int_0^T 2u'(t)y(t)dt \tag{9}$$

Let us observe for later use that (8) imply

$$\begin{bmatrix} j(x)+j'(x) & h(x)-\tfrac{1}{2}g'(x)\nabla P(x) \\ [h(x)-\tfrac{1}{2}g'(x)\nabla P(x)]' & -f'(x)\nabla P(x) \end{bmatrix} = \begin{bmatrix} w'(x) \\ \ell'(x) \end{bmatrix} [w(x) \quad \ell(x)] \geq 0 \tag{10}$$

It is also possible to define a lossless system by specializing (4) and (5) somewhat, and to obtain a corresponding specialization of (8). Thus we say (4) is lossless if (a) it is passive and (b) if $x(t_0) = x(t_1) = 0$, then

$$\int_{t_0}^{t_1} u'y \, dt = 0 \tag{11}$$

for all $u(\cdot)$. In this case, the results of [5] show that (7) and (8) hold with $\ell(x)$ and $w(x)$ both zero, and the matrix on the left side of (10) is accordingly zero.

3. SYNTHESIS PROCEDURE

For a single nonlinear inductor carrying current i and flux ϕ, assumed to have a ϕ-i characteristic passing through the origin, the stored energy is $\int_0^{\phi} i(\phi)d\phi$. For n mutually coupled nonlinear inductors with current vector i and flux vector ϕ, the stored energy is $\int_0^{\phi} i'(\phi)d\phi$; since this integral must be path-independent for a lossless set of inductors, $i(\phi)$ must be of the form $\nabla Q(\phi)$ for some scalar function Q of ϕ, nonnegative on account of the passivity property.

In our problem, we identify the state variable x with the vector of inductor fluxes, and the function $P(x)$ with the stored energy. Since $P(x) \geq 0$ for all x, this means that the inductors are certainly passive, indeed lossless. The current corresponding to the flux vector x is $\frac{1}{2}\nabla P(x)$. [In abstract terms, one may regard the inductor simply as the map $x \mapsto \frac{1}{2}\nabla P(x)$].

In Figure 1, we may evidently identify the variables v and i as $\dot{x}$ and $\frac{-\nabla P(x)}{2}$ respectively. Now observing (4) we see that the only way the coupling network could provide the requisite relation between u and y is if it sustains precisely the voltage-current pairs.

$$\begin{bmatrix} y \\ \dot{x} \end{bmatrix}, \quad \begin{bmatrix} u \\ -\frac{1}{2}\nabla P(x) \end{bmatrix}$$

or

$$\begin{bmatrix} h(x)+j(x)u \\ f(x)+g(x)u \end{bmatrix}, \quad \begin{bmatrix} u \\ -\frac{1}{2}\nabla P(x) \end{bmatrix}$$

These pairs in effect define the coupling network.

Note that in the event that the map $x \longmapsto \nabla P(x)$ is invertible, the coupling network will be current controlled (i.e. any current can exist at its ports) as will the coupled inductors. Further the coupling network is plainly nondynamic. If $x \longmapsto \nabla P(x)$ is not invertible, the network is still nondynamic, though controlled by something external to it, viz. the vector $\dot{x}$ of inductor fluxes.

Let us now observe the passivity of the coupling network. The instantaneous power flow into the network is

$$[u'j'(x)+h'(x) \quad u'g'(x)+f'(x)] \begin{bmatrix} u \\ -\frac{1}{2}\nabla P(x) \end{bmatrix}$$

$$= [u \quad 1] \begin{bmatrix} j(x)+j'(x) & h(x)-\frac{1}{2}g'(x)\nabla P(x) \\ [h(x)-\frac{1}{2}g'(x)\nabla P(x)]' & -f'(x)\nabla P(x) \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix}$$

$$\geq 0$$

using (10). Moreover, in case (4) is lossless, we know that $\ell(x)$ and $w(x)$ in (8) are zero, and use of (10) then implies here that the instantaneous power flow into the coupling network is zero. Hence if (4) is passive, so is the coupling network and if (4) is lossless, so is the coupling network.

4. CONCLUSIONS

The main result of this paper is the demonstration that a class of passive systems can be viewed as a cascade of a memoryless passive system (termed earlier the coupling network) and a dynamic lossless system (termed earlier the coupled inductor network). Some immediate variations on this theme are clearly possible; for example, one could work with admittances and capacitors, or one could exhibit a synthesis starting from an analogue of the scattering matrix. [Results akin to those of [5] have been developed by one of the authors which handle this problem].

Perhaps of more interest would be an examination of the extent to which reciprocity ideas could be incorporated into the study. Presumably one would parallel some of the linear system ideas used in [4], but the details remain unclear.

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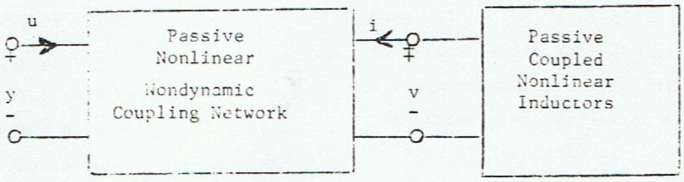

FIGURE 1

Cascade Decomposition of Nonlinear Network