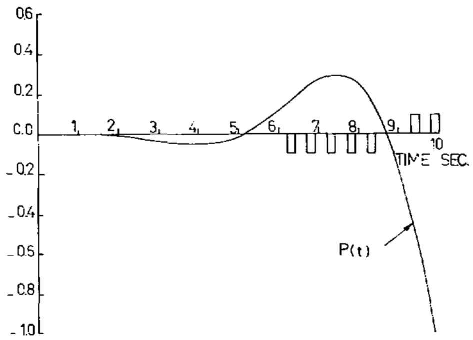

Fig. 2.

TABLE I

Pulse Number	Time of Pulse Initiation (sec) and Pulse Polarities in Parentheses		
	Case 1	Case 2	Case 3
1	0.000 (−)	0.000 (−)	6.221 (−)
2	0.500 (−)	0.500 (−)	6.721 (−)
3	1.000 (−)	1.000 (−)	7.221 (−)
4	1.806 (+)	1.800 (+)	7.721 (−)
5	2.306 (+)	2.300 (+)	8.221 (−)
6	2.806 (+)	2.800 (+)	9.300 (+)
7	3.306 (+)	3.300 (+)	9.800 (+)
8	3.806 (+)	3.800 (+)	
9	4.306 (+)	4.300 (+)	
10	4.806 (+)	4.800 (+)	
11	5.434 (−)	5.430 (−)	
12	5.934 (−)	5.930 (−)	
13	6.434 (−)	6.430 (−)	
14	6.934 (−)	6.930 (−)	
15	7.434 (−)	7.430 (−)	
16	7.934 (−)	7.930 (−)	
17	8.434 (−)	8.430 (−)	
18	8.800 (+)	9.300 (+)	
19	9.300 (+)	9.800 (+)	
20	9.800 (+)		

Case 3: A cost is applied on pulses and α is equal to 0.1, Fig. 2. The polarity and time of initiation of pulses constituting the optimal control $u(t)$ for these cases are given in Table I.

It is noticed that, in Case 1, the dead time between the pulses 17 and 18 is less than the allowable minimum dead time, while in Case 2, the dead time between any to adjacent pulses is equal to or greater than the required minimum. In Case 3, the spacing between pulses of opposite polarities is relatively large because the cost involved prohibits pulses when the magnitude of $P(t)$ is small; consequently, it is irrelevant whether or not a dead time is required between adjacent pulses of different polarities.

The values of $I^*(u)$ obtained for the three cases are -0.0581, -0.0568, and -0.0359, respectively. Note that the extemum of $I^*(u)$ is decreasing as constraints on the control function are added.

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On-Line Steady-State Control of a Synchronous Generator

PETER J. WORTLEY AND PETER J. MOYLAN

***Abstract*—Excitation control of a synchronous generator is discussed. Linear systems theory is used to derive a simple feedback formula.**

In any attempt to apply optimal control theory to a physical system, the first step is to obtain the state equations. For a synchronous generator these equations are quite complex with the result that the optimal controller is also likely to be quite complex. This note describes a method by which linear optimal control theory may be applied to the problem of machine control, in such a way that a relatively simple controller is derived.

A mathematical model for a salient pole synchronous generator was derived by Park [1] and subsequently used by others [2],[3]. The description is by nonlinear differential equations. In order to be able to apply results from linear systems theory, these equations can be linearized for small disturbances about a given operating condition. The equations depend of course on the operating point. Such an approach is useful for the steady-state stability analysis of the machine where only small changes from the operating point need be considered.

If we consider the system operating under steady-state conditions with constant mechanical input power (i.e., there are no small changes in input power) then the machine can be described about that operating point by a time invariant linear 3×3 matrix system relating a change in the field voltage (Δv_{fd}) as input to changes in the state variables torque angle ($\Delta\delta$), time derivative of torque angle ($\Delta\dot{\delta}$), and terminal voltage (Δv_t). Under these conditions the state equations can be derived and are given by

$$\dot{x} = Fx + Gu$$

where

$$x = \begin{bmatrix} \Delta\delta \\ \Delta\dot{\delta} \\ \Delta v_t \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 & 0 \\ C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \end{bmatrix}; \quad G = \begin{bmatrix} 0 \\ 0 \\ C_7 \end{bmatrix}.$$

C_1, C_3, C_4, C_5, and C_7 are dependent on the point about which linearization is performed. C_2 and C_6 are constant for a particular machine.

Optimal control theory for linear systems can be applied to the linearized system. This requires the solution of a matrix Riccati equation to determine the optimal feedback gains. However, if the machine is to be controlled in this way under all operating conditions the variations in the F and G matrices necessitate solving the Riccati equation at each operating point.

Such a process is difficult to accomplish on-line in either a digital system or an analogue system, mainly because the Riccati equation requires more time to solve than can be reasonably allowed.

One solution to this problem is to store the gains for every operating point of the machine [4]. Below, we indicate a simpler method.

If the state variable x is subjected to a time invariant transformation $\hat{x} = Tx$, where

$$T = \frac{1}{C_3 C_7} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

Manuscript received August 28, 1973; revised December 17, 1973. This work was supported in part by the Shortland County Council and in part by the Australian Research Grants Committee.
The authors are with the Department of Electrical Engineering, University of Newcastle, New South Wales, Australia.

then the system is put into canonical form

$$\dot{\hat{x}} = \hat{F}\hat{x} + \hat{G}\Delta v_{fd}$$

where

$$\hat{x} = Tx = \begin{bmatrix} \frac{\Delta\delta}{C_3C_7} \\ \frac{\Delta\dot{\delta}}{C_3C_7} \\ \frac{\Delta\ddot{\delta}}{C_3C_7} \end{bmatrix}$$

$$\hat{F} = TFT^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix}$$

$$\hat{G} = TG = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and

$$\begin{aligned} \alpha_1 &= C_1C_6 - C_3C_4 \\ \alpha_2 &= C_2C_6 - C_3C_5 - C_1 \\ \alpha_3 &= -(C_2 + C_6). \end{aligned}$$

Here, α_1 and α_2 are functions of the operating point (although they are constant at any given operating point), and α_3 is a constant.

When a system is in canonical form it is then possible to stipulate the eigenvalues of the stabilized system [5] by noting that the closed-loop system matrix, $\hat{F}_c$ is given by

$$\hat{F}_c = \hat{F} - \hat{G}\hat{K}$$

where K is the feedback gain matrix.

By arbitrarily choosing an initial set of feedback constants $\hat{K}_1$ such that

$$\hat{K}_1 = (-\alpha_1, -\alpha_2, 0)$$

then the closed-loop system matrix is given by

$$\hat{F}_{c_1} = \hat{F} - \hat{G}(-\alpha_1, -\alpha_2, 0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\alpha_3 \end{bmatrix}.$$

This intermediate system is completely controllable, completely observable, and has constant eigenvalues $0, 0, -\alpha_3$, independent of operating point.

Stabilization of this system is straightforward and can be achieved either by solution of a matrix Riccati equation or by simple choice of eigenvalues to yield a further feedback gain matrix $\hat{K}_2$ given by

$$\hat{K}_2 = (k_1, k_2, k_3).$$

$\hat{K}_2$ will be constant for all operating points of the machine and will in effect determine the response of the machine.

Thus a closed-loop system in canonical form is obtained which is known to be stable.

$$\hat{F}_c = \hat{F}_{c_1} - \hat{G}\hat{K}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -(\alpha_3 + k_3) \end{bmatrix}.$$

Relating these results to the original system by performing an inverse transformation yields a feedback gain matrix K and a closed-loop system matrix F_c given by

$$K = \hat{K}T = \left[\frac{(k_1 - \alpha_1) + C_1k_3}{C_3C_7}, \frac{(k_2 - \alpha_2) + C_2k_3}{C_3C_7}, \frac{k_3}{C_7}\right]$$

$$F_c = [F - GK] = T^{-1}\hat{F}_cT = \begin{bmatrix} 0 & 1 & 0 \\ C_1 & C_2 & C_3 \\ C_4 - \frac{(k_1 - \alpha_1) + C_1k_3}{C_3} & C_5 - \frac{(k_2 - \alpha_2) + C_2k_3}{C_3} & C_6 - k_3 \end{bmatrix}.$$

The eigenvalues of F_c are of course the same as those of $\hat{F}_c$ so the system is stable.

K is dependent on the operating point of the machine but can be hardware implemented and thus used as an on-line stabilizing feedback law for the machine.

Note that the final system may not be "optimal" in the normal sense. However, stability is ensured and the type of response desired can be stipulated.

Further it is not necessary to use the state vector as given above for the machine description since all descriptions should reduce to the same canonical form by a suitable linear transformation T.

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Eigenvalue Control in Distributed-Parameter Systems Using Boundary Inputs

SPYROS G. TZAFESTAS

***Abstract*—The feedback eigenvalue control problem of distributed-parameter systems subject to boundary inputs is solved and results are derived for both single-eigenvalue and multi-eigenvalue assignment. An illustrative example is included.**

I. Introduction

The eigenvalue control problem has received much attention in recent years [1]–[5]. The majority of the results concern the case of lumped-parameter systems. Our aim here is to solve the eigenvalue control problem of distributed-parameter systems from their boundaries. The approach is based on the Green's identity. The results are useful since in most cases the control of practical distributed-parameter systems is effected from the boundary surfaces rather than the interior of the occupied spatial domains. The theory is illustrated by means of a simple example.

II. The Boundary Eigenvalue Control Problem

Given a scalar system defined over an n-dimensional spatial domain D with boundary surface ϑD,

$$\frac{\vartheta X(x,t)}{\vartheta t} = A_x X(x,t), \qquad x \in D, \quad t \geq t_0 \tag{1}$$

Manuscript received February 20, 1974. This note is a revised form of the main results presented at the International Conference on Systems and Control, PSG College, Coimbatore, India.
The author is with the Department of Electrical Engineering, University of Patras, Patras, Greece, and with the N.R.C. Democritos, Aghia Paraskevi Attikis, Athens, Greece.